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education

Department:
Education
REPUBLIC OF SOUTH AFRICA

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APRIL 2010

NATIONAL CERTIFICATE

MATHEMATICS N4

(16030164)

29 March (X-Paper)
09:00 – 12:00

Calculators may be used.

This question paper consists of 5 pages and a 1-page formula sheet.

DEPARTMENT OF EDUCATION
REPUBLIC OF SOUTH AFRICA
NATIONAL CERTIFICATE
MATHEMATICS N4
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Answer ALL the questions in full. Show ALL the calculations and intermediary steps. Simplify where possible.
4. ALL the graph work must be done in the ANSWER BOOK. Graph paper is NOT supplied. Values of intercepts with the system of axes and the turning point(s) MUST be shown on the graph.
5. ALL final answers must be accurately approximated to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must NOT be separated.
7. A formula sheet is attached to this question paper. You are NOT compelled to use the formulae and the list is NOT necessarily complete.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 The difference between two numbers is 7. The sum of their squares is 29. Find the numbers. (5)

1.2 Solve for x if:

$5\left(3^{2x+2}\right)=31$ (3)

1.3 Given: $i=I\left(e^{\frac{v}{x}}+1\right)$

Make g the subject of the formula. (4)

1.4 Given:

$3y+2x=z+1$

$3x+2z=8-5y$

$3z-1=x-2y$

Solve for y by using Cramer's rule. (8)

[20]

QUESTION 2

2.1 2.1.1 Sketch the graph of $\frac{x^2}{36}-\frac{y^2}{49}=1$. (3)

2.1.2 What is the domain of the graph of: $\frac{x^2}{36}-\frac{y^2}{49}=1$ (1)

2.2 2.2.1 Sketch the graph of $y=x^2+5$. (3)

2.2.2 Is the graph of $y=x^2+5$ symmetric about the y-axis? (1)

2.3 Sketch the graph of $x^2+y^2=12{,}25$. (2)

2.4 Solve for x and y if:

$(x+y)+j(x-y)=14{,}8$ and $j6{,}2$ (4)

2.5 Given: $Z=-3+j2$

2.5.1 Convert Z into polar form. Show ALL the steps. θ may only be positive. (3)

2.5.2 Represent Z and ALL calculated values on the Argand diagram. (3)

[20]

QUESTION 3

3.1 Prove that:

$\text{cosec } B \cdot \cot B + \text{cosec}^2 B = \dfrac{1}{1-\cos B}$ (3)

3.2 Solve for x if:

$4\sin^2 x - 5\cos x = 2\ ;\ 0^0 \leq x \leq 360^0$ (4)

3.3 If $\sin x = \dfrac{12}{13}$ and $\cos y = \dfrac{8}{17}$ and both x and y are acute angles, calculate WITHOUT the use of a calculator, the value of $\sin(x+y)$. (4)

3.4 Derive a formula for $\cos 2B$ in terms of $\sin B$. (3)

3.5 Calculate WITHOUT the use of a calculator, the value of $\text{cosec } 75^0$. (3)

3.6 Prove that $\cos 2A = \dfrac{7}{25}$ if $\sin A = \dfrac{3}{5}$. (3)

[20]

QUESTION 4

4.1 Differentiate from the first principles if $y = x^2 - 3x$. (4)

4.2 Differentiate the following with respect to x:

$y = \dfrac{\cos 2x}{\cos x + \sin x} - \dfrac{6}{\sqrt{x}} - \dfrac{1}{10e^x} + \dfrac{4}{\cos x}$ (5)

4.3 Differentiate with the aid of Quotient rule if: -

$y = \dfrac{\sin x}{\sec x}$ (4)

4.4 Given:

$y = x^3 - 6x^2 + 11x - 6$

Calculate, with the aid of differentiation, the co-ordinates of the minimum and the maximum turning points. Distinguish between the minimum and the maximum turning points by using the second derivative. (7)

[20]

QUESTION 5

5.1 Integrate the following:

$$\int\left(8e^{-4t}-3\cos 3t+\frac{1}{\sqrt{t}}-11^{2t}+3\sqrt{t}+2\sec^2 t\right)dt$$ (7)

5.2 Evaluate the following:

$$\int_2^3 \frac{dt}{t}$$ (3)

5.3 Simplify the following:

$$\int\sqrt{\cos^2 x+\sin^2 x}\quad dx$$ (3)

5.4 5.4.1 Sketch and indicate the area enclosed by the graph of $y = 4x$ with $x=2$ and $x=3$. Also indicate the representative strip to be used to calculate the area enclosed. (3)

5.4.2 Calculate, using integration, the area indicated in QUESTION 5.4.1. (4)

[20]

TOTAL: 100

MATHEMATICS N4

FORMULA SHEET

NEW SYLLABUS

$a^x = b \Leftrightarrow \log a^x = \log b$

$\ell n\, x = \log_e x$

$(r\,|\underline{\theta})^n = r^n\,|\underline{n\theta}$ $\quad a + bj = c + dj \Leftrightarrow a = c$ and $b = d$

$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$
$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\sin^2 x + \cos^2 x = 1$
$1 + \cot^2 x = \mathrm{cosec}^2 x$
$1 + \tan^2 x = \sec^2 x$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

y	$\frac{dy}{dx}$
ax^n	nax^{n-1}
ka^x	$ka^x \ell na$
$k\ell nx$	$\frac{k}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

$$y = u(x) \cdot v(x)$$

$$\Rightarrow \frac{dy}{dx} = u(x)v'(x) + u'(x)\,v(x)$$

$$y = \frac{u(x)}{v(x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C$$

$$\int \frac{a}{x}\,dx = a\ell n\,x + c$$

$$\int ka^x\,dx = \frac{ka^x}{\ell na} + c$$

$$A_{ox} = \int_a^b y\,dx$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \tan x\,dx = \ell n \sec x + c$$

$$\int \sec x\,dx = \ell n(\sec x + \tan x) + c$$